DISSERTATION PROPOSAL

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"Network flow-based approaches for Binary Linear Bilevel Optimization"

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10:30am
Tepper 4242

Bilevel optimization models hierarchical decision-making problems involving two agents, a leader and a follower, who interact while optimizing separate objective functions subject to shared and individual constraints. Such models naturally arise in transportation network design, management science, supply chain management, finance, and machine learning. When the follower's problem contains only continuous variables, strong duality, Karush-Kuhn-Tucker conditions, or variational inequalities may be used to obtain single-level reformulations that can be handled by off-the-shelf optimization solvers. However, when the follower's problem involves discrete decisions, the resulting problem becomes particularly challenging: standard relaxations are weak, the follower's value function is nonconvex, and most techniques from the continuous case no longer apply. Despite substantial progress in recent years, current state-of-the-art methods can handle only small to medium-sized instances and often fail to exploit the combinatorial structure of the follower's problem.

This dissertation develops a new methodology for binary linear bilevel optimization that leverages network flow representations to construct stronger relaxations and more scalable reformulations. The overarching goal is to represent or approximate the follower's value function through encodings that preserve a combinatorial structure and enable efficient computation.

Chapter 1 lays the theoretical foundation by studying optimistic binary linear bilevel programs (BBPs), where in the presence of multiple optima, the follower cooperatively selects a response favorable to the leader. We propose a novel single-level nonlinear reformulation of optimistic BBPs using a decision diagram encoding of the follower's feasible region. This approach yields a family of scalable relaxations by restricting the decision diagram and provides dual bounds that can be iteratively strengthened to optimality. We also extend the framework to the pessimistic variant, in which the follower acts adversarially. Computational experiments demonstrate competitive performance with state-of-the-art solvers on a set of benchmark instances—closing two previously unsolved instances and improving six known upper bounds—and a superior performance on a set of instances where the follower's problem exposes a combinatorial structure amenable to a decision diagram encoding. This work, coauthored with Leonardo Lozano and Willem-Jan van Hoeve, has been accepted for publication in Mathematical Programming.

Chapter 2 builds on this foundation by introducing new network flow-based single-level linear reformulations of BBPs that operate directly in the original variable space. While the reformulation in Chapter 1 may be strong, it can also be computationally expensive to solve. To address this, we propose using the underlying network structure as an oracle for generating valid cuts within a Branch-and-Cut algorithm, and to strengthen valid cuts through a tilting procedure. On computationally generated instances with large or multiple networks, this approach outperforms the monolithic reformulation, and the tilting procedure reduces both the number of required cuts and the size of the Branch-and-Bound tree by roughly two orders of magnitude. This is ongoing joint work with Leonardo Lozano and Willem-Jan van Hoeve.

Chapter 3 extends these ideas to Mixed-Integer Generalized Nash Equilibrium Problems (MIGNEPs), which can, in certain cases, be viewed as BBPs with multiple non-cooperative followers. These problems arise in applications such as network design, tolling systems, and scheduling, and are intrinsically challenging due to the interdependence of the players' feasible sets and objectives. While existing methods address specific MIGNEP applications, the only general-purpose approach is that of Duguet et al. (2025), which leverages integer bilevel programming techniques. Building on our developments for BBPs, we adapt the single-level reformulations to this multi-follower setting and show how information sharing across network representations can lead to stronger bounds and more efficient algorithms. This ongoing work is in collaboration with Leonardo Lozano and Willem-Jan van Hoeve.

Chapter 4, motivated by applications in Public-Private Partnerships, seeks to bridge stochastic and bilevel programming. Existing approaches often impose restrictive assumptions on the players' decision timeline—for example, allowing only the follower to act after uncertainty is revealed—which may not align with these applications. We propose a more general framework in which both the leader and follower solve mixed-integer two-stage stochastic programs. We develop new decomposition-amenable reformulations that integrate ideas from both literatures—including results from previous chapters—enabling more scalable solution methods. Preliminary computational experiments on two problem classes illustrate the effectiveness of the approach and its advantages over existing methods, which already struggle to solve small-sized instances. This work is in collaboration with Merve Bodur and Bernardo Pagnoncelli.

Overall, this dissertation contributes new theoretical insights, reformulations, and computational techniques for binary bilevel optimization. It advances our understanding of how structured network encodings can be systematically exploited to improve tractability and scalability in hierarchical decision-making problems.

Proposed Committee: Willem-Jan van Hoeve (Chair), Leonardo Lozano, John Hooker, Merve Bodur, Ted Ralphs